

# Linking Sphere Topology in M-Theory and Its Relation to SFIT

Douglas G. Stevenson  
stevensonfluxinformationtheory.com

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## 1 Introduction

Linking sphere topology is the key topological mechanism that enforces charge quantization for extended objects in higher-dimensional gauge theories, including M-theory. It generalizes the familiar Dirac monopole quantization to higher-form gauge fields and higher-dimensional branes.

This document explains the concept rigorously, derives the linking condition for the M2-brane, and shows how it connects to the effective flux quantization in Stevenson-Flux Information Theory (SFIT).

## 2 General Definition of Linking

In an  $n$ -dimensional manifold  $X$ , two closed oriented submanifolds  $A^p$  (dimension  $p$ ) and  $B^q$  (dimension  $q$ ) are said to be **\*\*linked\*\*** if they cannot be continuously deformed away from each other without intersecting, and their dimensions satisfy

$$p + q + 1 = n.$$

The **\*\*linking number\*\***  $\text{Lk}(A, B)$  is a topological invariant that counts how many times  $B$  winds around  $A$ .

In gauge theory, if  $A$  is a source for a  $(p + 1)$ -form gauge field, the linking sphere  $B$  (of dimension  $q = n - p - 1$ ) measures the enclosed flux:

$$Q = \int_B \star F_{p+2}.$$

This integral must be quantized for consistency of the quantum theory.

### 3 Dirac Monopole Example (4D)

- Source: magnetic monopole (0-dimensional point). - Linking sphere:  $S^2$  (2-sphere) in 3 spatial dimensions. - Gauge field: 1-form  $A$  (electromagnetic vector potential). - Flux:  $\int_{S^2} F = 2\pi n/e$ .

The topology forces  $eg = 2\pi n$ .

### 4 M2-Brane in 11D Spacetime

- M2-brane worldvolume dimension: 3 (2 spatial + time). - Spacetime dimension:  $n = 11$ . - Transverse dimension:  $11 - 3 = 8$ . - Linking sphere:  $S^7$  (7-dimensional sphere) embedded in the 8 transverse directions.

The M2-brane is electrically charged under the 3-form  $C_3$ , with field strength  $F_4 = dC_3$ . The linking sphere  $S^7$  measures the electric flux:

$$Q_2 = \int_{S^7} \star F_4.$$

Any continuous deformation of this  $S^7$  that tries to unlink it must intersect the M2-brane worldvolume, making the linking topologically non-trivial.

### 5 Mathematical Derivation of Quantization

Under a large gauge transformation  $C_3 \rightarrow C_3 + d\Lambda_2$ , the Wess-Zumino term changes by

$$\Delta S_{WZ} = T_2 \int_{\Sigma_3} d\Lambda_2 = T_2 \int_{S^7} \star F_4,$$

where  $\Sigma_3$  is the M2-brane worldvolume.

For the path-integral phase factor to be single-valued,

$$e^{iT_2 \int_{S^7} \star F_4} = e^{2\pi i n}, \quad n \in \mathbb{Z}.$$

With the M2-brane tension  $T_2 = (2\pi)^{-2} \ell_{11}^{-3}$ , this gives the quantization condition

$$\int_{S^7} \star F_4 = 2\pi n \ell_{11}^3, \quad n \in \mathbb{Z}.$$

The linking  $S^7$  topology is what forces the flux to be discrete.

### 6 Why the Linking Sphere Cannot Be Contracted

The  $S^7$  is embedded in the complement of the M2-brane worldvolume. Because the worldvolume has codimension 8 in 11D spacetime, the complement has the homotopy type that prevents the  $S^7$  from being continuously shrunk to a point without crossing the brane. This non-trivial homotopy enforces the integer quantization.

## 7 Connection to SFIT Flux Quantization

In M-theory, linking sphere topology ( $S^7$ ) enforces quantized flux at the Planck scale. In SFIT, the analogous topology appears at laboratory scales. The “linking cycle” is the closed phase-space orbit of the ultra-cold neutron in the gravitational potential. The resonant information flux at  $\nu_{\text{res}} = 1.20134$  mHz plays the role of the higher-form gauge field.

The single-valuedness of the neutron wave function under this flux leads to the SFIT quantization condition

$$\Phi_{\text{total}} = n \cdot \frac{h\nu_{\text{res}}}{K}, \quad n \in \mathbb{Z},$$

with  $K = 1.060$ .

Thus, the Planck-scale topological quantization of M2-brane flux manifests at low energies as the measurable resonant information flux in SFIT.

## 8 Conclusion

Linking sphere topology is the mechanism that forces charge quantization in higher-dimensional gauge theories. For the M2-brane in 11D, an  $S^7$  links the 3-dimensional worldvolume. This same topological principle underlies the effective flux quantization in SFIT at laboratory scales.

## References

## References

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